Practical Considerations when Estimating in the Presence of Autocorrelation

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When correcting for autocorrelation, most econometrics texts suggest using a quasi-differencing procedure. A number of issues arise. First, it is found that the results from popular two-step procedures may differ dramatically from those obtained from iterative processes. Second, while it is true that most regression packages implement an iterative procedure, the methodology itself is not conveyed in a straightforward manner to students of econometrics. Third, given the various iterative methods in the literature, it is not always clear which method is superior. Fourth, for autocorrelated errors, the importance of the correction factor in simple forecasting is often overlooked. Finally, regression packages report an R^2 that is not comparable to that from the Ordinary Least Squares (OLS) estimation. This paper succinctly outlines the procedure for performing iterative procedures, explicitly accounts for autocorrelation among errors when generating forecasts, and identifies the necessary transformations for making proper comparisons relating to R^2.

1. Introduction

Students of econometrics learn that when the errors from a linear regression are correlated, the problem of serial correlation or autocorrelation arises. If autocorrelation is ignored, the Ordinary Least Squares (OLS) estimators remain unbiased, consistent, and asymptotically normally distributed; however, these estimators are no longer efficient and the standard errors are biased, generally downwards. As a consequence, the usual t, F, and χ^2 tests are no longer valid. In order to correct for autocorrelation, one often uses generalized least square (GLS) procedures such as the Cochrane-Orcutt or Prais-Winsten two-step or iterative procedures, which are

based on a particular estimator for the correlation coefficient (Greene, 2003; Gujarati, 2003; Ramanathan, 2002; Stock and Watson, 2003; Wooldridge, 2003).

This paper has a number of objectives. First, practitioners should exercise caution if choosing to use two-step procedures when correcting for autocorrelation. The application used in this paper shows that these methods may produce results that differ from those obtained using iterative procedures. Second, while it is true that seasoned econometricians understand the iterative process and most regression packages implement

an iterative procedure automatically, the methodology itself is not conveyed in a straightforward manner to econometric students. Third, given a choice of various methodologies available when estimating a model with first-order autocorrelation, a natural question is to ask which of these methods is superior. This paper discusses these issues, offers some suggestions, and succinctly outlines the procedure for performing iterative procedures.

In addition, while the theory of forecasting with the presence of autocorrelation has been well documented, econometric students often have difficulty formally applying this technique. The process of making a dynamic forecast – a forecast that accounts for errors made in past forecasts – is clearly delineated. Finally, the majority of regression packages report statistics, such as R^2, that are not directly comparable to the OLS ones. The proper adjustments that are necessary for valid comparisons are identified.

Section 2 addresses the above methodological issues and Section 3 implements the procedures empirically. Section 4 concludes.

2. The Generalized Least Square (GLS) Procedure

Here it is assumed that the error term in the linear regression model is identically, but not necessarily independently distributed. The model is

$$Y_t = \beta_1 + \beta_2 X_{2,t} + ... + \beta_k X_{k,t} + \varepsilon_t, \tag{1}$$

$$\varepsilon_t = \rho\varepsilon_{t-1} + v_t, \quad -1 < \rho < 1 \tag{2}$$

where, although the v_t are independently and identically distributed (i.i.d.), the ε_t are not for $\rho \neq 0$. This is the simplest case of autocorrelation, called first-order autocorrelation.[1]

If ρ is known, the method of generalized differencing is applied to the model to transform it into one in which the errors are independent. In order to describe the procedure briefly, one uses the fact that the linear model in Eq. (1) holds for all time periods, thus

$$Y_{t-1} = \beta_1 + \beta_2 X_{2,t-1} + ... + \beta_k X_{k,t-1} + \varepsilon_{t-1}. \tag{3}$$

Multiplying Eq. (3) by ρ and subtracting from Eq. (1), yields the following transformation:

$$Y_t^* = \beta_1^* + \beta_2^* X_{2,t}^* + ... + \beta_k^* X_{k,t}^* + v_t, \tag{4}$$

where $Y_t^* = Y_t - \rho Y_{t-1}$,

$\beta_1^* = \beta_1(1 - \rho)$,

$\beta_j^* = \beta_j$ for all $j \neq 1$,

$X_{jt}^* = X_{jt} - \rho X_{jt-1}$, and

$v_t = \varepsilon_t - \rho\varepsilon_{t-1}$.

Since by construction the transformed equation, Eq. (4), has a modified error that is i.i.d., the resulting parameter estimators are efficient. Thus, applying OLS to Eq. (4) yields the following estimated GLS regression:[2]

$$\tilde{Y}_t^* = \tilde{\beta}_1^* + \tilde{\beta}_2^* X_{2,t}^* + ... + \tilde{\beta}_k^* X_{k,t}^*. \tag{5}$$

The generalized differencing procedure is useful if the value of ρ is known *a priori*. Since this usually is not the case, the estimated GLS is based on an estimate of ρ which is often obtained by the Cochrane-Orcutt or Prais-Winsten procedures. Under the assumption of normality, these procedures produce estimators that converge to the same probability limit as the maximum likelihood estimators, which are consistent and asymptotically efficient.

The Cochrane-Orcutt procedure is implemented as follows:

Step 1. The method of Ordinary Least Squares is used to estimate Eq. (1).

Step 2. The residuals from Eq. (1) are $\hat{\varepsilon}_t = Y_t - \hat{Y}_t$, where $\hat{Y}_t = \hat{\beta}_1 + \hat{\beta}_2 X_{2,t} + ... + \hat{\beta}_k X_{k,t}$. These residuals are regressed against their lagged values to obtain $\hat{\rho}$.

Step 3. Eq. (4) is estimated with $\hat{\rho}$ used in place of ρ.

If the estimation process stops after Step 3, then this is generally referred to as the Cochrane-Orcutt two-step procedure. The estimates for the β's and their standard errors from Step 3 are now the valid estimates of the parameters in Eq. (1). Most texts note that the estimate for the intercept from Step 3 must be adjusted by dividing it by $1 - \hat{\rho}$. This same adjustment, however, must also be made to the standard error of the intercept.

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[1] The GLS procedure can be easily extended to deal with higher-order autoregressive schemes. It should also be noted that this procedure is valid under the conditions that the explanatory variables are strictly exogenous.

[2] The hat symbol denotes estimation with OLS, whereas the tilde symbol denotes estimation with GLS.

The Cochrane-Orcutt two-step procedure provides only a single estimate of ρ. The Cochrane-Orcutt iterative procedure estimates ρ iteratively, that is, by successive approximation, starting with some initial value of ρ. In order to apply the Cochrane-Orcutt iterative procedure, one proceeds from Step 3 to Step 4:

Step 4. The estimates for the β's from Step 3 ($\tilde{\beta}$) are then used to obtain a new set of errors.
It is at this juncture where the confusion lies: The errors should be calculated using

$$\tilde{\varepsilon} = Y_t - \tilde{Y}_t, \qquad (6)$$

where $\tilde{Y}_t = \tilde{\beta}_1 + \tilde{\beta}_2 X_{2,t} + ... + \tilde{\beta}_k X_{k,t}$, $\tilde{\beta}_1 = \dfrac{\tilde{\beta}_1^*}{(1-\hat{\rho})}$ [3] and

$\tilde{\beta}_j = \tilde{\beta}_j^*$. Students commonly make a mistake in computing the errors as $Y_t^* - \tilde{Y}_t^*$ from Eq. (4). In fact, this step has not been explicitly stated in any of the texts cited earlier.

Step 5. Step 2 is then repeated. The general recommendation is to stop carrying out iterations when the successive estimates of ρ differ by a small amount, say, by less than 0.0001.

When the Cochrane-Orcutt two-step or iterative procedure is applied, the differencing procedure results in one lost observation since the first observation has no antecedent. In small samples it has been documented that keeping the first observation or omitting it can make a substantial difference in the regression results. The loss of one observation in large samples tends to be inconsequential. The Prais-Winsten transformation is a procedure that preserves the first observation on Y and X_j as given by $Y_1\sqrt{1-\hat{\rho}^2}$ and $X_{j,1}\sqrt{1-\hat{\rho}^2}$. In the application that follows, results will be produced using both the Cochrane-Orcutt procedure as well as applying the Prais-Winsten transformation.

Both Cochrane-Orcutt and Prais-Winsten methods are based on the estimate $\hat{\rho}$. Although $\hat{\rho}$ is commonly derived by running a regression of the residuals on their lagged value (as in Step 2 above), there are various other ways to estimate it. For instance, since the Durbin Watson (DW) statistic can be approximated by *2(1-ρ)*, another convenient estimator is derived as

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[3] When Gujarati (p. 493) outlines the steps for conducting the iterative Cochrane-Orcutt procedure in an exercise, he too, overlooks the necessary adjustment to the intercept.

$\hat{\rho}_{DW} = 1 - \dfrac{DW}{2}$. Theil-Nagar and others have provided modifications for finite samples.

There are many issues of practical importance regarding the choice between various methods of estimating a first-order autoregressive model. A practitioner has to choose between (a) the two-step and the iterative methods of estimation, (b) Cochrane-Orcutt and Prais-Winsten estimation procedures, and (c) various estimators for ρ used in the GLS estimation of the model. A natural question is to ask which of these methods is superior. As it turns out, all of the above methods employed in the GLS estimation have the same asymptotic distribution. Further, since their small sample properties are difficult to derive, researchers have resorted to Monte Carlo experiments for direction.

As is often the case, the evidence from Monte Carlo studies is by no means conclusive and is only indicative at best. We outline the general suggestions as follows (see also Greene, 2003, p.274): (1) For small samples, with ρ less than 0.3, OLS is actually preferred to GLS since it avoids the additional sampling variation caused by the extra estimator $\hat{\rho}$; (2) it is preferable to employ the iterative method over the two-step method even if the marginal gain is minimal; (3) the Prais-Winsten approach is superior despite the simplicity of the Cochrane-Orcutt method achieved by dropping the first observation; and (4) the regression of residuals on their lagged values to estimate $\hat{\rho}$ (as in Step 2 above) generally works well.

We would like to point out that iterative procedures suffer from the possible problem of converging to a local, and not necessarily the global, minimum. The OLS regression in Step 1 is based on the default starting value of zero for $\hat{\rho}$. This value is subsequently updated over the iterations. It is advisable to implement this iterative procedure with several different starting values of ρ to ensure that the convergence takes place at the same parameter vector. If the estimators oscillate, the search method proposed by Hildreth-Lu is suggested (see for example, Greene 2003), despite its computational complexity. The idea is to form a grid of ρ values in small steps between (-1, 1) and search for the value that minimizes the residual sum of squares from the corresponding transformed equations for the GLS estimation. All of the above estimation procedures are readily available in most statistical packages including SPSS, SAS, and STATA.

Further, as mentioned earlier, if the autocorrelation among the residuals in a regression model is ignored and

the OLS procedure is used to estimate the parameters, then the forecasts will be unbiased since the estimates are unbiased. However, these forecasts will be inefficient with larger variances. Employing the Cochrane-Orcutt or Prais-Winsten procedures will result in estimates that are both unbiased and efficient. Efficient forecasts are produced by explicitly accounting for the autocorrelation among the residuals. Substituting $T+1$ for t into Eq. (5) yields the following:

$$\tilde{Y}_{T+1}^* = \beta_1^* + \tilde{\beta}_2^* X_{2,T+1}^* + \ldots + \tilde{\beta}_k^* X_{k,T+1}^*, \tag{7}$$

where T represents the number of observations in the sample, $\tilde{Y}_{T+1}^* = \tilde{Y}_{T+1} - \tilde{\rho} Y_T$, and $X_{j,T+1}^* = X_{j,T+1}^o - \tilde{\rho} X_{j,T}$. The value $X_{j,T+1}^o$ is some known (or estimated) value of X_j for the time period, $T+1$.

Rearranging and solving for $\tilde{Y}_{T+1}$ yields the following one-period forecast:

$$\tilde{Y}_{T+1} = \tilde{\beta}_1 + \tilde{\beta}_2 X_{2,T+1}^o + \ldots + \tilde{\beta}_k X_{k,T+1}^o + \tilde{\rho}\tilde{\varepsilon}_T \tag{8}$$

where $\tilde{\varepsilon}_T = Y_T - \tilde{\beta}_1 - \tilde{\beta}_2 X_{2,T} + \ldots + \tilde{\beta}_k X_{k,T}$. Following this logic, the forecast for Y_{T+s} is obtained as:

$$\tilde{Y}_{T+s} = \tilde{\beta}_1 + \tilde{\beta}_2 X_{2,T+1}^o + \ldots + \tilde{\beta}_k X_{k,T+1}^o + \tilde{\rho}^s \tilde{\varepsilon}_T \tag{9}$$

Students of econometrics often ignore the correction factor, $\tilde{\rho}^s \tilde{\varepsilon}_T$. A forecast using Eq.(8) will be made in section 3.

It is true that most statistical packages carry out the Cochrane-Orcutt or Prais-Winsten iterative procedures with a simple command, thus relieving the student of the tedious iteration process. However, many widely used programs such as SPSS, EVIEWS, and STATA, report an R^2 that is not comparable to the corresponding OLS estimates. Given the knowledge that OLS maximizes R^2, it would be constructive to obtain a comparable R^2 from alternative methods. A suitable way of calculating R^2, for purposes of comparison with the original OLS results, can be calculated as follows: $R^2 = \left(correlation(Y_t, \tilde{Y}_t)\right)^2$ where Y_t represents the original values of the dependent variable and $\tilde{Y}_t = \tilde{\beta}_1 + \tilde{\beta}_2 X_{2,t} + \ldots + \tilde{\beta}_k X_{k,t}$. Many packages erroneously calculate R^2 as $R^2 = \left(correlation(Y_t^*, \tilde{Y}_t^*)\right)^2$ where $Y_t^* = Y_t - \tilde{\rho} Y_{t-1}$ is the transformed value of Y_t.

3. An Application

We illustrate the above GLS methods using an application that analyzes the relationship between the inflation rate, the deficit, and the short-term interest rate. The following regression model is estimated:[4]

$$rate_t = \beta_0 + \beta_1 inflation_t + \beta_2 deficit_t + \varepsilon_t, \tag{10}$$

where *rate* is the annualized three-month T-bill interest rate, *inflation* is the annual inflation rate based on the consumer price index (CPI), and *deficit* is the federal budget deficit as a percentage of GDP. It is common in the literature to use the three-month T-bill interest rate as a proxy for the short-term interest rate. Most studies also use the federal budget deficit rather than the total budget deficit in their analysis (see, for example, Hoelscher, 1983, Makin, 1983, and Mascaro and Meltzer, 1983). Data from 1948 to 1996 are obtained from the *2004 Economic Report of the President* and are presented in Table 1.

Inflation and the Interest Rate

Inflation is the rate of change in the average level of prices. Generally, prices tend to rise faster when the economy is operating near its peak and they tend to fall, or at least rise less rapidly, when the economy is near a trough. The interest rate is also procyclical in that it tends to rise during recovery periods and fall during recessions. Nominal interest rates generally rise with inflation to compensate lenders for the falling purchasing power of the dollar. If the public's anticipation of inflation is instantaneously met, then a one percentage point increase in the inflation rate is likely to cause a one percentage point increase in the interest rate. If the actual inflation rate is different from the anticipated inflation rate, then the relationship between the inflation rate and the interest rate is still expected to be positive, however not one-to-one.

The Deficit and the Interest Rate

The federal budget deficit is calculated as federal government tax revenues minus outlays. When the government runs a deficit, it must borrow from the public. The financing of this is generally in the form of interest-bearing bonds. As the government sells bonds to the public to finance the deficit, borrowers now must compete for a reduced pool of loanable funds. In order to

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[4] This is an extension of an example used in Jeffrey Wooldridge's *Introductory Econometrics: A Modern Approach* (pp. 336-37). However, the example is not applied in an autocorrelation context in Wooldridge's text.

arrive at a new equilibrium in the loanable funds market, the price of money – the interest rate – tends to rise. Thus, on theoretical grounds, increases in government borrowing cause the interest rate to rise. However, proponents of the total crowding-out effect, a rather extreme position, argue that the rise in the interest rate causes a reduction in private investment spending which in turn may eventually lead to a fall in the interest rate that counters its initial rise. In addition, empirical support linking the positive correlation between the deficit and the interest rate has been weak at best (see Hoelscher (1986) and references therein). The conflicting results are not surprising when one reasons that the potential problem of simultaneity is likely present when estimating the relationship between the deficit and the interest rate; that is, it is just as reasonable to theorize that a higher interest rate would tend to raise government outlays, thereby increasing the deficit.

Column 2 of Table 2 presents the OLS regression results. Ignoring for the moment the fact that autocorrelation is problematic, the OLS results suggest that increases in the inflation rate and the relative size of the deficit increase the short-term interest rate. The results indicate that although there appears to be a positive relationship between inflation and the interest rate, this relationship is not on a one-to-one basis. For instance, a one percentage point increase in inflation, holding the relative size of the deficit constant, increases interest rates, on average, by 0.610 percentage points. The initial results pointing to a positive relationship between the deficit and the interest rate also challenge the theory put forth by crowding-out proponents. Both independent variables are significant at the 5% level. The coefficient of determination, R^2, indicates that 70% of the variability in the interest rate is explained by the variability in the independent variables. However, a Durbin-Watson statistic of 0.927 indicates that positive autocorrelation exists.

Columns 3 and 5 of Table 2 present the two-step Cochrane-Orcutt and Prais-Winsten results. The two-step results of both models suggest that inflation and

Table 1. Three-month T-Bill rate, Consumer Price Index and Federal Budget Deficit as a Percentage of Gross Domestic Product, United States, 1948-1996

Year	Three-month T-Bill interest rate, annualized	Percentage Annual Change in the Consumer Price Index	Federal Budget Deficit as a Percentage of GDP	Year	Three-month T-Bill interest rate, annualized	Percentage Annual Change in the Consumer Price Index	Federal Budget Deficit as a Percentage of GDP
1948	1.04	8.1	-4.6	1973	7.041	6.2	1.1
1949	1.102	-1.2	-0.2	1974	7.886	11.0	0.4
1950	1.218	1.3	1.1	1975	5.838	9.1	3.4
1951	1.552	7.9	-1.9	1976	4.989	5.8	4.2
1952	1.766	1.9	0.4	1977	5.265	6.5	2.7
1953	1.931	0.8	1.7	1978	7.221	7.6	2.7
1954	0.953	0.7	0.3	1979	10.041	11.3	1.6
1955	1.753	-0.4	0.8	1980	11.506	13.5	2.7
1956	2.658	1.5	-0.9	1981	14.029	10.3	2.6
1957	3.267	3.3	-0.8	1982	10.686	6.2	4.0
1958	1.839	2.8	0.6	1983	8.63	3.2	6.0
1959	3.405	0.7	2.6	1984	9.58	4.3	4.8
1960	2.928	1.7	-0.1	1985	7.48	3.6	5.1
1961	2.378	1.0	0.6	1986	5.98	1.9	5.0
1962	2.778	1.0	1.3	1987	5.82	3.6	3.2
1963	3.157	1.3	0.8	1988	6.69	4.1	3.1
1964	3.549	1.3	0.9	1989	8.12	4.8	2.8
1965	3.954	1.6	0.2	1990	7.51	5.4	3.9
1966	4.881	2.9	0.5	1991	5.42	4.2	4.5
1967	4.321	3.1	1.1	1992	3.45	3.0	4.7
1968	5.339	4.2	2.9	1993	3.02	3.0	3.9
1969	6.677	5.5	-0.3	1994	4.29	2.6	2.9
1970	6.458	5.7	0.3	1995	5.51	2.8	2.2
1971	4.348	4.4	2.1	1996	5.02	3.0	1.4
1972	4.071	3.2	2.0	1997	5.07	2.3	0.3

Source: *2004 Economic Report of the President.*

the relative size of the deficit still have a positive and significant effect on the interest rate.[5] As suspected when positive autocorrelation exists, the magnitude of the standard errors of the coefficients does increase once the models are corrected for autocorrelation; yet this increase in the standard errors is not enough to change the significance of the independent variables. Similarly, and as expected, the two-step Cochrane-Orcutt and Prais-Winsten procedures generate slightly smaller values of R^2 of 67.3% and 69.8%, respectively. It should also be noted that both two-step procedures produce Durbin-Watson statistics that fall in the indeterminate region.

Table 2. Regression Results of the Effects of Inflation and Deficits on Interest Rates

Dependent Variable: The Three-month T-bill interest rate, annualized (1948-1996)

	OLS	Two-Step CO	Iterative CO	Two-Step PW	Iterative PW
Constant	1.252*[a]	2.596*	4.994*	2.093*	3.509*
	(0.439)[b]	(0.673)	(1.321)	(0.648)	(1.143)
Inflation	0.610*	0.486*	0.220*	0.484*	0.255*
	(0.075)	(0.086)	(0.097)	(0.089)	(0.100)
Deficit	0.710*	0.353*	-0.061	0.513*	0.089
	(0.118)	(0.157)	(0.171)	(0.141)	(0.166)
R^2	0.700[c]	0.673	0.342	0.698	0.591
Durbin-Watson	0.927	1.478[e]	1.772	1.575	1.791
$\hat{\rho}$	n/a[d]	0.523	0.841	0.523	0.830

Notes: (a) * denotes significance at the 5% level.
(b) Standard errors are in parentheses; in addition, standard errors for the constant have been properly adjusted.
(c) R^2 is defined in terms of the T-bill interest rate (non-modified) for all models.
(d) n/a denotes 'not applicable.'
(e) Durbin-Watson statistic is estimated for the quasi-differenced dependent variable.

The dramatic change in the results occurs, however, when the iterative Cochrane-Orcutt and Prais-Winsten procedures are applied. Columns 4 and 6 of Table 2 demonstrate that inflation is still positively and significantly related to the interest rate. However, both the Cochrane-Orcutt and Prais-Winsten results indicate that the relative size of the deficit now has an insignificant influence on the interest rate. The deficit's lack of impact on the interest rate appears to support arguments put forth by crowding-out proponents. The value of R^2 has declined sharply to 34.2% for the

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[5] A regression of the residuals on their lagged value (as in Step 2) is used in these regressions. For the sake of brevity, other methods to estimate ρ are not included in the tables.

Cochrane-Orcutt process and it drops less drastically, to 59.1%, for the Prais-Winsten process. Finally, the estimates of ρ in the final iteration are also quite different from the estimates of ρ in the two-step processes. The two-step and iterative Cochrane-Orcutt procedures produce estimates of ρ of 0.523 and 0.841, respectively. Similarly, the two-step and iterative Prais-Winsten procedures yield estimates of ρ of 0.523 and 0.830, respectively.

Table 3. Estimates of ρ during each iteration

Iteration	$\hat{\rho}$ under CO	$\hat{\rho}$ under PW
1	0.5225	0.5225
2	0.6674	0.6323
3	0.7622	0.7031
4	0.8130	0.7538
5	0.8325	0.7876
6	0.8386	0.8078
7	0.8405	0.8188
8	0.8410	0.8244
9	0.8412	0.8272
10	0.8412	0.8285
11	0.8413	0.8292
12	0.8413	0.8295
13	0.8413	0.8297
14	0.8413	0.8297
15	0.8413	0.8298

Table 3 shows the estimates of ρ during each iteration. Most authors downplay the need for iterations. For instance, Greene notes that "since the estimator is asymptotically efficient at every iteration, nothing is gained by doing so" (p. 273). However, in this basic application, the choice of the estimate of ρ is of great importance. For instance, using an estimate of 0.523 in the Prais-Winsten process yields the result that the relative size of the deficit has a positive and significant effect on the interest rate. This same variable has an insignificant affect on the interest rate when an estimate of 0.830 is employed. Further, the regression model appears to have relatively good explanatory power when an estimate of 0.523 is used for ρ, yielding an R^2 of 69.8%; yet, when ρ is estimated with a value of 0.830, the calculated R^2 drops to 59.1%.

At this point, it would be useful to generate forecasts using Eq. (8) for illustration. Since the data in the application covers the time period 1948-1996, values for the independent variables (inflation and the deficit) are available for the year 1997. For known values of inflation and the deficit for the year 1997, the following 1997 forecast is calculated using the Cochrane-Orcutt iterative estimates:

$rate_{1997} = 4.994 + (0.220 * infl_{1997}) - (0.061 * deficit_{1997}) + (0.841 * \tilde{\varepsilon}_{1996})$
$= 4.994 + (0.220 * 2.3) - (0.061 * 0.3) + (0.841 * -0.546)$
$= 5.02$

Similarly, the 1997 forecast using the Prais-Winsten iterative estimates yields:

$rate_{1997} = 3.509 + (0.255 * infl_{1997}) + (0.089 * deficit_{1997}) + (0.830 * \tilde{\varepsilon}_{1996})$
$= 3.509 + (0.255 * 2.3) + (0.089 * 0.3) + (0.830 * 0.622)$
$= 4.64$

As mentioned earlier, these forecasts explicitly account for the autocorrelation among errors. Interestingly, the actual value for the interest rate in 1997 was 5.07; if the last expressions in the equations had been ignored, (that is, omitting ' $+0.841*\tilde{\varepsilon}_{1996}$ ' in the Cochrane-Orcutt estimation and ' $+0.830*\tilde{\varepsilon}_{1996}$ ' in the Prais-Winsten estimation), then the Cochrane-Orcutt and Prais-Winsten estimates for 1997 would have been 5.48 and 4.12, respectively.

4. Conclusion

Autocorrelation is a topic that is addressed in virtually every econometrics class. It is shown here that the results obtained from the Cochrane-Orcutt or Prais-Winsten two-step procedures may differ from those obtained from iterative procedures. A survey of relevant econometric texts also reveals that the Cochrane-Orcutt and Prais-Winsten iterative procedures are not outlined in a straightforward manner. This paper delineates the iterative procedure in a clear and systematic way. By summarizing the results of Monte Carlo studies, it also offers some guidance to a practitioner who has to choose between various estimation methods. Further, one-step-ahead forecasts are generated that explicitly account for the nature of the autocorrelation. Finally, the proper value of R^2 is derived to make this statistic comparable to the one obtained from the OLS method.

We would like to point out that the above empirical analysis is based on the sometimes unrealistic assumption that the explanatory variables are strictly exogenous. It is possible that there are feedbacks from this year's interest rate into next year's deficit and/or inflation rate. Under weaker contemporary exogeneity assumptions, OLS still provides consistent estimators while iterative GLS fails to do so. Perhaps, as occurred in this application, a drastic change in the estimates from OLS, to two-step, to iterative methods is indicative of a violation in the exogeneity assumption. If this is the case, an attractive alternative is to use the OLS estimates with a correction for the standard errors using the Newey–West method (see, for example,Wooldridge 2003).

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